their nidification presents nothing that may be deemed peculiar or even specially characteristic. In their flight and manner of procuring their food, however, they differ strikingly from all other birds, in these respects closely resembling certain insects, especially the crepuscular hawkmoths (Sphingidæ). Their food, consisting mainly of small insects, but in part also of the nectar of flowers, is mostly gleaned from blossoms, before which they poise, with wings so rapidly vibrating as to be invisible except as a dim haze or halo partly surrounding the body and producing the humming sound from which these birds derive their vernacular name, the bill thrust inside the flower and the slender, semitubular tongue extended into the depths of the blossom. Some species, instead of feeding from flowers, glean their insect food from the bark of forest trees, following along the branches in suspended flight in the same manner that the others pass from flower to flower. In their feeding from flower to flower, Humming Birds, like bees, butterflies, and moths, perform the same office in the economy of nature as insects by transferring pollen from one bloom to another, and thus assisting in the fertilization of plants. In flying from one point to another, the flight of Humming Birds, while essentially direct, is usually more or less undulating, and so extremely rapid that the eye can scarcely follow. Often this flight is accompanied (at least in the case of males of some species) by a more or less remarkable screeching or grating sound, produced mechanically by some peculiarity of wing-structure.

"Diminutiveness of size and metallic brilliancy of coloring are the chief external characteristics of Humming Birds, though exceptions to both occur; and in these respects they, as a group, have no rivals. Unfortunately, stuffed specimens convey but a faint idea of their splendid coloring, for the perfection of their changeable refulgence can be fully realized only in the living bird, whose every change of position flashes to view a different hue—emerald green replacing ruby red, sapphire blue succeeding fiery orange, or either becoming opaque velvety black—according to the angle at which the sun's rays touch the feathers, an effect which can only partially be imitated with the stuffed specimen by artificially changing its position with reference to the light. Many species have a spot of the most luminous or brilliantly metallic color (usually green) that it is possible to imagine on the forehead at the base of the bill, this spot being surrounded by the most intense velvety black—evidently to enhance the brilliancy of the ornament by contrast, just as a jeweler would, for the same purpose, display a diamond or other gem against a background of black velvet. Often there is a spot of brilliant color and one of a contrasting hue just below it, the result being that first one color, then the other, is flashed forth as the bird changes slightly its position."

The thirty-one plates give the structural details of bill, wings, tail and feet of each of the 121 genera, thus greatly facilitating identification. It is hoped that Part V. may be followed in due time by the remaining volumes of this invaluable work, so indispensable to all students of American birds.

J. A. ALLEN

The Hindu-Arabic Numerals. By DAVID EUGENE SMITH and LOUIS CHARLES KARPINSKI. Boston and London, Ginn and Company. 1911. Pp. vi + 160.

This book gives in compact form a readable and carefully prepared account of the numerous researches which have been made in the endeavor to trace the origin and development of the Hindu-Arabic numerals. Teachers of mathematics will welcome it, while students specializing in the history of mathematics will derive great help from the many bibliographical references to other publications on this subject. Like the arithmetician Tonstall the authors read everything in every language and spent much time in licking what they found into shape *ad ursi exemplum,* as the bear does her cubs. But it would not be a correct statement, were we to convey the idea that the book does not embody original research. In several cases the authors have been able to correct mistakes of earlier writers

and to add results of their own research. In a few instances this history appears to us incomplete and defective. This we shall endeavor to show in what follows.

The authors very properly give much attention to the study of routes of commercial travel. There is every reason to believe that the migrations of the numerals took place along commercial routes. The authors consider the possibility of an early influence of China upon India; they speak of trade routes and the interchange of thought by land and sea, between countries bordering on the Mediterranean and far-off India. They even point out early relations of Greece with China. In view of these careful studies it is singular that practically nothing should be said on the intercourse which did or did not exist between Babylonia and India during the centuries immediately preceding and following the beginning of the Christian era. They ignored a question which lies at the root of present-day speculations on the earliest traces of the principle of local value and the symbolism for zero. Of course, local value is considered by the authors in connection with the Hindu-Arabic numerals. Not to do so would be to examine the shell and ignore the kernel. Were these fundamental notions wholly of Hindu origin or were the rudimentary ideas relating to them imported into India from neighboring territory? In the book under review this vital question is not adequately discussed. The authors are correct in stating that the preponderance of authority has been in favor of the hypothesis that our numeral system, with its concept of local value and our symbol for zero has been of Hindu origin. But this conclusion is coming to be recognized as unsafe. The change of opinion that is taking place is voiced by two German authors of brief histories, Tropfke and Günther. In 1902 Tropfke said[1]

> Dass unser Positionssystem mit seinen Ziffern indischen Ursprunges ist, steht fest.

[1] "Geschichte der Elementar-Mathematik," I., p. 10.

In 1908 Günther said[2]

> Man kann . . . sich den Vorgang vielleicht so denken, dass Indien von Babylon her die ersten schwachen Andeutungen des Stellenwertes empfing, sie in seiner Weise um- und ausbildete und später das reiche Geschenk des fertigen Positionssystemes den Nachkommen jener Geber zurückerstattete.

The evidence in favor of a possible Babylonian origin is even stronger than as stated by Günther, for he was apparently unaware that symbols for zero had been found in Babylonia. These symbols are mentioned, but not adequately discussed by Smith and Karpinski, on page 51. The facts in our possession to-day are about as follows:

1. Two early Babylonian tablets, one probably dating from 1600 or 2300 B.C., use the sexagesimal system and the all-important principle of local value. It so happens that they contain no number in which there was occasion to use a zero.

2. Babylonian records from the centuries immediately preceding the Christian era give a symbol for zero which was apparently "not used in calculation." It consisted of two angular marks, one above the other, roughly resembling two dots, hastily written.

3. About A.D. 130, Ptolemy in Alexandria used in his "Almagest," the Babylonian sexagesimal fractions and also the omicron O to represent blanks in the sexagesimal numbers. The symbol was not used as a regular zero.

4. Strabo and others have described the trade routes by land and the trade between Babylonia and India, also the trade by sea.

5. Sexagesimal fractions were used by Hindu astronomers. Historians do not deny that the Indian sexagesimal fractions were of Babylonian origin.

6. The earliest form of the Indian symbol for zero was that of a dot, which, according to Bühler, was "commonly used in inscriptions and manuscripts in order to mark a blank." This early use of the symbol resembles somewhat the still earlier use made of symbols for zero by the Babylonians and by Ptolemy. Probably Āryabhata, in the fifth century

[2] "Geschichte der Mathematik," I., p. 17.

A.D., knew our zero. The earliest undoubted occurrence of our zero in India is in 876 A.D.

Were there overflows of Babylonian science into Greece and India? The question is pertinent. The possibility of overflows into India has been recognized not only by Günther, but by Nallino, who states that the Chaldeans of 100 B.C. (and even earlier) knew the sidereal year (estimated at 365 d. 6 hr. 13 m. 43 sec.) and that this knowledge probably passed from them to the Hindus and Persians. This statement is quoted by H. Suter with apparent approval.[3] It seems to us that these facts point directly toward a summary of the case, somewhat as quoted above from Günther.

In recounting the earliest uses made of the Arabic numerals in Egypt and the Occident, reference is made to a trace on a pillar of a church in Egypt, giving the date 349 A.H. (= A.D. 961). Strange to say our authors completely ignore similar evidence, as given in the *Philosophical Transactions* of London. Why should this be? No less prominent a mathematician than John Wallis arrived at the conclusion that "their use in these parts was as old at least as . . . the middle of the eleventh century."[4] Wallis refers to the "Mantle-tree of the Parlour Chimney at the dwelling House of Mr. Will. Richards, the Rector of Helmdon in Northampton-shire," bearing an inscription with the date "A° Do[i] M° 133" (= A.D 1133). Thomas Luffkin[5] names a building in Colchester bearing the date 1090. These dates are of interest to an Englishman or an American. If they are to be rejected, it would seem that the reasons therefor should be set forth in a publication aiming to weigh minutely all respectable evidence. Smith and Karpinski make no mention of an earlier use than 1539 of the numerals in Great Britain.

In describing the different shapes of the zero in Europe the authors overlooked the curious use of *theta* θ to represent zero, found in the writings of Michel Rolle, and Eneström's note on this notation in "L'Intermédiaire des mathématiciens," II., 1895, p. 283.

It has been said of early American geologists that they crossed the western plains, eager to reach the Rocky Mountains, there to grapple with the problems relating to the geology of our land, that in so doing they neglected the geologic problems presented by the plains themselves. In the same way our authors hastened back half a thousand years to reach the conspicuously formative periods, and in so doing they forgot to take note of matters of interest connected with recent time, which for historical research is far from barren. Our authors note the different shapes which the numerals assumed among the Hindus, Arabs and Europeans of the Renaissance. But if I mistake not, interesting forms, worthy of study, are found in seventeenth and eighteenth century manuscripts, stored away in American libraries. It is of some interest that the figure 8 sometimes had the shape of our dollar mark written with a single downward stroke, thus, $. I remember mistaking such an eight for a dollar mark and recognizing my error only when the sum given in the manuscript would not come out as represented, except on the supposition that the mark stood for 8. The authors point out anomalous combinations of the Hindu-Arabic and the Roman numeral symbols which occurred more or less accidentally in the fifteenth and sixteenth centuries, but they fail to notice a curious combination which occurs with surprising regularity in Spanish-American manuscripts during the three centuries preceding the beginning of the nineteenth century. Of this notation we shall speak in a separate article.

On page 28, line 10, the word "vertical" should be replaced by the word "horizontal." In the table of contents there is an omission such that the pronoun "their" in the line "Early ideas of their origin" refers to

[3] "Bibliotheca Mathematica," Vol. 5_3, 1904, p. 85.

[4] *Philosophical Transactions*, No. 154, p. 399 (= Abridgment, Vol. I., 1705, p. 107).

[5] *Philosophical Trans. Abridged*, Vol. I., 1705, p. 108.

"oriental names"; it should refer to "Hindu-Arabic numerals." The alphabetical index is not as complete as one might wish it to be.

FLORIAN CAJORI
COLORADO COLLEGE,
COLORADO SPRINGS, COLO.

A Laboratory Course in Physiology. By WALTER B. CANNON, A.M., M.D., George Higginson professor of physiology in the Harvard Medical School. Second edition. Published by Harvard University. 1911.

This is the set of loose-leaf laboratory notes and directions used in the course in physiology in the Harvard Medical School. It belongs to a class of works which have only begun to appear in recent years. It is not a general laboratory manual like the well-known handbook of Burdon Sanderson, or that of Stirling. Its scope is much narrower. While these works aimed to give, within the limits of their size, accounts of all ordinary physiological methods, the work before us, on the contrary, is merely a precise description of a particular course. Accordingly, it is limited to such methods as the facilities of the Harvard School allow. Within these limitations, however, it is excellent. It has already been adopted as the basis of the physiological course in a number of other institutions and contains much that is valuable and suggestive for the teaching of physiology anywhere.

The most striking defect of this "course" is that it contains far too much of the physiology of the frog and too little of the mammal. For the medical student direct personal experience in working with the circulation in one living cat or dog is worth two or three experiments upon the frog's heart, and a dozen upon the frog's leg. It is most unfortunate that the limitations which misguided humanitarians and anti-vivisectionists place upon the supply of cats in Boston should make it necessary to have the circulation in this animal worked out by the students in *groups of twelve.* This certainly falls far short of the important educational principle urged by Pearce that "the students should do it themselves." The reviewer knows from personal experience that the largest number of students who can possibly take part in a blood-pressure experiment on one cat is five. If mammalian material were as abundant as it ought to be for such a course, the work on the frog here outlined could profitably be cut in half. Each group should number four or five students instead of twelve and should have, instead of one cat, six to ten.

Much of the work on the frog here given could be profitably replaced by experiments on man. Simple sphygmomanometers can be provided cheaply, and should be used for experiments on the students themselves on a much more extensive scale than is outlined in these notes.

The weakest point in the notes is the section on respiration. Only eight pages are devoted to this subject, while muscle nerve physiology receives eighteen. The progress in knowledge of respiration within recent years, for which we are indebted principally to Haldane and his pupils, has been made largely by experiments upon man. These experiments are ideally suited to a laboratory course. Among them may be mentioned that of voluntary forced breathing and the succeeding apnœa; that of the artificial production of Cheyne-Stokes breathing requiring for its demonstration merely a tin of soda lime and a long tube; and that of the duration of the voluntary holding of the breath without preparation, after forced breathing, after oxygen and after forced breathing and oxygen.

These, however, are merely criticisms of detail. In general this work is certainly by far the best of its kind that has yet appeared. No other educational institution in America, perhaps none in the world, in recent years has made so many valuable experimental contributions to the theory and methods of teaching as has Harvard. Among these contributions not the least valuable is the demonstration that science in general and physiology in particular can be, and ought to be, taught by laboratory methods. Originally conceived by Huxley and first practised in this country by Newell Martin at Johns Hopkins and by the